May 2005, Vol.20, No.3, pp.289–295 *J. Comput. Sci. & Technol.*

Knowledge Map: Mathematical Model and Dynamic Behaviors

Hai Zhuge and Xiang-Feng Luo

China Knowledge Grid Research Group, Key Lab of Intelligent Information Processing
Institute of Computing Technology, Chinese Academy of Sciences, Beijing 100080, P.R. China

E-mail: zhuge@ict.ac.cn, luoxf@kg.ict.ac.cn

Received September 27, 2004; revised March 11, 2005.

Abstract Knowledge representation and reasoning is a key issue of the Knowledge Grid. This paper proposes a Knowledge Map (KM) model for representing and reasoning causal knowledge as an overlay in the Knowledge Grid. It extends Fuzzy Cognitive Maps (FCMs) to represent and reason not only simple cause-effect relations, but also time-delay causal relations, conditional probabilistic causal relations and sequential relations. The mathematical model and dynamic behaviors of KM are presented. Experiments show that, under certain conditions, the dynamic behaviors of KM can translate between different states. Knowing this condition, experts can control or modify the constructed KM while its dynamic behaviors do not accord with their expectation. Simulations and applications show that KM is more powerful and natural than FCM in emulating real world.

Keywords knowledge representation, causal relation, fuzzy cognitive maps, knowledge grid

1 Introduction

Fuzzy cognitive maps, aiming to mimic human causal knowledge reasoning[1,2], have the following characteristics[1−6]:

1) knowledge is in the form of concepts and directional connections;
2) relation between concepts is in the form of production rule;
3) reasoning can be computed by numeric matrix operations;
4) tacit knowledge of experts can be expressed;
5) data structure of its knowledge representation (adjacency matrix) is addable.

FCMs have been widely used in economy, fault analysis[7], information systems[8,9], tacit knowledge management[10], industry control[11,12], virtual world[5,13], electronic commerce[14] and so on. Although FCMs have many desirable properties, they have the following limitations[15].

1) Unable to reason on sequential relations or time-delay causal relations existing widely in real world. The characteristic of their reasoning process is that all the interaction of FCMs' concepts is synchronous.

2) Unable to reason on conditional probabilistic causal relations.

Fig.1 is an FCM representing an expert's knowledge about terror events. The concept *casualty* and the *explosion* are the subsequences of terrorists, and the *terrorists* is the subsequence of *the friendly foreign policy* and *the power of strike. The friendly foreign policy* does not have an immediate effect and it needs days or months to make a full impact on *terrorists. The power of strike* also needs hours or days to exert a full impact on *terrorists*. The relation degree between *terrorists* and *explosion* is uncertain and closely related to the current states. In the normal state, the relation degree from *explosion* to *terrorists* is low, but in the war state, the degree from *explosion* to *terrorists* is high. FCMs do not support this type of reasoning, so their reasoning results in some intelligent systems are usually distorted.

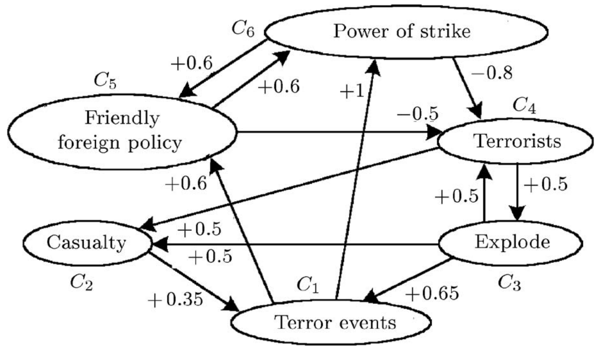

Fig.1. Expert's knowledge about terror events represented by fuzzy cognitive map.

Hagiwara extends FCMs to solve time-delay causal relations and conditional causal relations[4], but he does not consider probabilistic causal relations. Miao *et al.* put forward Dynamical Cognitive Network (DCN) to define dynamic causal relation between concepts[3], but DCN still does not distinguish different types of causal relations. Neural Cognitive Map (NCM) is to solve complex causal relations by Obata and Hagiwara[16], but NCM needs much training data that are difficult to be obtained in some intelligent systems, further, time-delay causal relations and conditional probabilistic causal relations are difficult to be found and explained by neural networks. Based on the object-oriented paradigm for decision support, contextual Fuzzy Cognitive Map (cFCM)

*Regular Paper
This work was supported by the National Natural Science Foundation of China (Grant Nos.60273020, 60402016, and 70271007) and the National Basic Research 973 Program of China (Grant No.2003CB317000).

is proposed by Liu and Satur[8,9]. The cFCM considers contextual causal relations and simple cause-effect relations without probabilistic causal relations and time-delay causal relations.

Most of the extended Cognitive Maps (CMs) do not consider their dynamic behaviors and the fact that these CMs are manually constructed by experts. If the dynamic behaviors of the constructed CMs do not accord with the experts' expectation, the experts need some rules to help them control or modify the behaviors of these CMs.

This paper discusses the mathematical model and the dynamic behaviors of the proposed cognitive map and classifies the causal relations into four kinds: simple cause-effect relation, time-delay causal relation, conditional probabilistic causal relation, and sequential relation (a special time-delay casual relation).

2 Mathematical Model of Knowledge Map

Knowledge Map (KM) is a kind of extended cognitive maps that can represent and reason simple cause-effect relations, time-delay causal relations, conditional probabilistic causal relations and sequential relations. Its mathematical model is represented as follows:

$$V_{C_j}(t+1)=\phi\Big(\sum_{\substack{i=1\\ i\neq j}}^{N} g(V_{C_i}(t), r_{ij}, w_{ij}), \sum_{\substack{i=1\\ i\neq j}}^{N} V_{C_i}(t)w_{ij}\Big),$$

where V_{C_i} and V_{C_j} are the state values of C_i and C_j; r_{ij} and w_{ij} are the relation type and the weight from C_i to C_j respectively; $\phi(x)$ is the reasoning function of C_j. r_{ij} and w_{ij} are the elements of KM's relation type matrix $\boldsymbol{R}$ and the adjacency matrix $\boldsymbol{E}$ respectively.

The operator function $g(V_{C_i}(t), r_{ij}, w_{ij})$ is determined by the following Rules 1–4:

Rule 1. If there exist conditional probabilistic causal relations from concepts C_i to C_j, $V_{C_j}(t+1)$ should be computed first, and set $w_{ij}=0$. The function can be defined as follows:

$$V_{C_j}(t+1)=\begin{cases} V_{C_j}(t)+u\Big(\sum_{i=1}^{N} V_{C_i}(t)w_{ij}|C_p, C_q, \ldots\Big), \\ \qquad\qquad \text{if } -1 \leqslant V_{C_j}(t)+u\Big(\sum_{i=1}^{N} V_{C_i}(t)w_{ij}\Big) \leqslant 1 \\ 1, \qquad \text{if } V_{C_j}(t)+u\Big(\sum_{i=1}^{N} V_{C_i}(t)w_{ij}\Big) > 1 \\ -1, \qquad \text{if } V_{C_j}(t)+u\Big(\sum_{i=1}^{N} V_{C_i}(t)w_{ij}\Big) < -1 \end{cases}$$

where C_p, $C_q, \ldots$ are conditions of weight w_{ij}, $u(x)$ is a computing function of all the concepts C_i occurrences leading to the increase/decrease of the probability of the concept C_j.

$$u(|x|)=\begin{cases} +u(|x|), & \text{if } x \geqslant 0, \\ -u(|x|), & \text{if } x < 0, \end{cases}$$

where $u(|x|)=\tanh(|x|)\in[0,1]$, and $+u(|x|)$ and $-u(|x|)$ mean that the occurrence of the causal concepts C_i leads to the increase and decrease of the probability of the effect concept C_j respectively.

Rule 2. If there exists time-delay causal relation from C_i to C_j, reserve the primary value of C_i during the time delay.

Rule 3. If all values of the i-th row are zero in the KM's adjacency matrix $\boldsymbol{E}$, then $V_{C_i}(t)$ is equal to the original value of C_i.

Rule 4. The $(m-1)$-th sequential relation should be reasoned before the reasoning of the m-th sequential relation.

After the operations of Rules 1–4, reason each concept of KM by the following rule:

Rule 5. The effect concept's state value of KM at time $(t+1)$ partly depends on its own state value at time t[1]. The state value of the effect concept is computed as follows:

$$V_{C_j}(t+1)=\mu f\Big(\sum_i w_{ij}V_{C_i}(t)\Big)+\lambda V_{C_j}(t)\in[-1,1],$$

where $f(x)=\tanh(x)$, μ and λ are allotted coefficient, and $\mu+\lambda=1$.

In KM, causal knowledge is in the form of concepts, relations, directional connections and their weights. Concepts and their state values represent knowledge and their existent degrees respectively. The relation type matrix $\boldsymbol{R}$ and the adjacency matrix $\boldsymbol{E}$ of KM describe the relation types and the weights between the concepts of directional connections respectively. And all of their interactions among concepts, relations, directional connections and weights compose a dynamic network and form a track that corresponds to a flow in the causal relation space. The terror events knowledge represented by FCM (see Fig.1) can be described as the knowledge map (denoted as KM1) shown in Fig.2. The relation type matrix and the adjacency matrix of KM1 correspond to $\boldsymbol{R}1$ and $\boldsymbol{E}1$ respectively. In the relation matrix, the simple cause-effect relation and the conditional probabilistic causal relation are denoted as R_{sc} and $R_{P/condition}$ respectively. If there does not exist causal relation between concepts, it is denoted as N in $\boldsymbol{R}$. The m-th subsequence is denoted as mS and the time-delay causal relation is denoted as nD

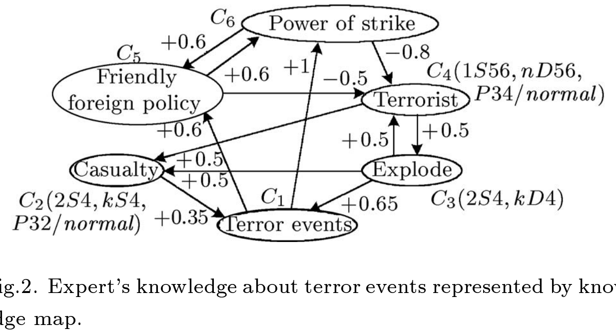

Fig.2. Expert's knowledge about terror events represented by knowledge map.

(n is the delay time). For example, $\boldsymbol{R}1(5,4) = (1S, nD)$ represents that there exists first sequential relation and the time-delay causal relation (delay time is n) from C_5 to C_4. The notation C_4 ($1S56$, $nD56$, $P34$/normal) in Fig.2 represents that there exists first sequential relation and the time-delay causal relation (delay time is n) from C_5 and C_6 to C_4, and there also exists conditional probabilistic causal relation from C_3 to C_4.

$$R1 = \begin{bmatrix} N & N & N & N & R_{sc} & R_{sc} \\ R_{sc} & N & N & N & N & N \\ R_{sc} & R_{P/normal} & N & R_P & N & N \\ N & (2S,kD) & (2S,kD) & N & N & N \\ N & N & N & (1S,nD) & N & R_{sc} \\ N & N & N & (1S,nD) & R_{sc} & N \end{bmatrix}$$

$$E1 = \begin{bmatrix} 0 & 0 & 0 & 0 & +0.6 & +1 \\ +0.35 & 0 & 0 & 0 & 0 & 0 \\ +0.63 & +0.5 & +0.5 & +0.5 & 0 & 0 \\ 0 & +0.5 & +0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5 & 0 & +0.6 \\ 0 & 0 & 0 & -0.8 & +0.6 & 0 \end{bmatrix}.$$

KM is a feedback dynamic system, the final state of the track of the interactions in the causal relations space may be fixed-points, limit-cycle, or chaotic/oscillatory behaviors. Simple KMs can achieve fixed-points or limit-cycle equilibrium behaviors. Complex KMs can achieve chaotic/oscillatory behaviors. Fixed-points mean that all the concepts' state values of KM are fixed and all the relations of knowledge are equilibria. Limit-cycle repeats a sequence of events or a chain of actions and responses. Chaos/oscillation represents that all the relations and concepts exist complex interactions, and we cannot predict their events, actions or responses[5]. If the above behaviors of the constructed KMs do not accord with the experts' expectation, the weights, and the relations or the concepts of the KM need to be adjusted. Therefore, the dynamic behaviors of KM should be investigated so as to guide the experts to control or modify the dynamic behaviors of the constructed KM.

If all the elements of the KM's relation type matrix $\boldsymbol{R}$ are the simple cause-effect relation, the KM retrogresses to FCM. Therefore, $g(V_{C_i}(t), r_{ij}, w_{ij})$ in KM retrogresses to $V_{C_i}(t) \times w_{ij}$. In this case, KM has the following characteristics:

(1) If $\sim C_i \in C$, $\sim V_i \in V$, $\sim r_{ij} \in R$ and $C_i \xrightarrow{w_{ij}, r_{ij}} C_j$ exist, then $C_i \xrightarrow{\sim w_{ij}, r_{ij}} \sim C_j$, $\sim C_i \xrightarrow{\sim w_{ij}, r_{ij}} C_j$, and $\sim C_i \xrightarrow{w_{ij}, r_{ij}} \sim C_j$ also exist, according to [9].

(2) If $r_{ij} = R_{P/condition}$, $\sim C_i \in C$, $\sim V_i \in V$, $\sim r_{ij}(t) \in R$, $P(w_{ij}(t)|V_{C_i}(t), \ldots)$ and $P(\sim w_{ij}(t)| \sim V_{C_i}(t), \ldots)$ exist, then $\sim w_{ij}(t) = 1 - w_{ij}(t)$.

Here C and V represent all the concepts and their values in KM, C_i, and $\sim C_i$ represent a causal concept and its opposition, C_j and $\sim C_j$ represent an effect concept and its opposition. For example, if C_i and C_j represent the fall of stock and raise of interest rate, then $\sim C_i$ and $\sim C_j$ represent the raise of stock and the fall of interest rate respectively.

3 Experiments on the Dynamic Behaviors of KMs

Miao and Liu[2] have proved that the problem of finding whether a state is reachable in the FCM is nondeterministic polynomial (NP) hard. So, it is difficult to analyze the final dynamic behaviors of KMs by mathematics. Here we study the dynamic behaviors of KMs based on experiments.

3.1 Approach for Measuring KM's Dynamic Behaviors

We define the following energy function to measure the disorder of the concepts' dynamic output of KM:

$$Energy_{process}(t) = -\frac{1}{2}\sum_{i=1}^{node} h\Big(\sum_j w_{ij} V_{C_j}(t)\Big) V_{C_i}(t),$$

where w_{ij} is the weight from C_i to C_j; $V_{C_j}(t)$ is the state value of concept C_j; $h(x)$ is a node function. $Energy_{process}$ can be regarded as an indicator of the dynamic behaviors of KM because it can measure the reasoning process of KM.

3.2 Weights Influencing the Dynamic Behavior of KMs

3.2.1 Weights Changing with Linearity

The relations and adjacency matrixes of the experimental KMs are randomly constructed by using Matlab programs. The weights of these KMs change according to:

$$w_{ij}(t+1) = w_{ij}(t) + decay \times w_{ij}(t), \tag{1}$$

where *decay* is the decay factor on the changed speed of $w_{ij}(t)$.

Fig.3 shows the experimental results of the dynamic behaviors of KMs while the weights are varied with (1). (Please refer to the inside back cover for Fig.3.) This experiment indicates the following results that can be used to guide the experts to control or modify the dynamic behaviors of the constructed KMs while the weights are varied with linearity:

1) starting with the equilibrium behaviors of fixed-points, the behavior can change to limit-cycle equilibrium behaviors while the decay is around 0^+;

2) starting with the fixed-points or the limit-cycle equilibrium behaviors, these behaviors can change to chaotic/oscillatory behaviors while the decay is less than -2;

3) starting with the chaotic/oscillatory behaviors, they can change to the limit-cycle equilibrium behaviors while the decay is around 0^+;

4) the behaviors of 0 states can be achieved while the decay in the interval $[-2, 0^+]$ regardless the starting point.

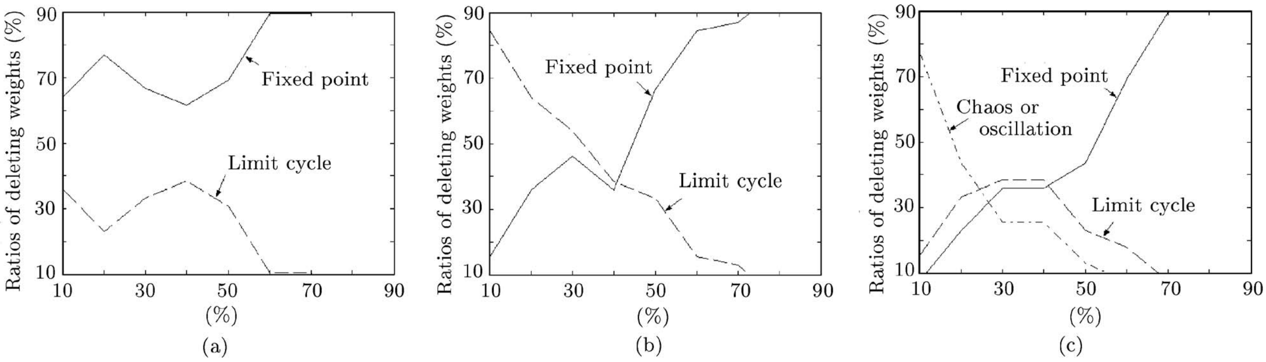

Fig.5. Dynamic behaviors of KMs while weights are deleted from 10% to 90%. (a) Dynamic behaviors of KMs starting with fixed-points equilibrium behaviors. (b) Dynamic behaviors of KMs starting with limit-cycle equilibrium behaviors. (c) Dynamic behaviors of KMs starting with chaotic/oscillatory behaviors.

3.2.2 Weights Changing with Nonlinearity

In KMs, the weights change with nonlinearity as follows:

$$w_{ij}(t+1) = decay * w_{ij}(t)^2 \tag{2}$$

Fig.4 shows the results of the dynamic behaviors of KMs while weights change with (2). (Please refer to the inside back cover for Fig.4.) According to Fig.4, the following conclusions can be achieved to guide the experts to control or modify the dynamic behaviors of the constructed KMs while the weights change with (2).

1) the fixed-points equilibrium behaviors can be achieved while the decay is bigger than 2 regardless the starting point;

2) the behavior of 0 states can be achieved while the decay is in the interval $[-1.5, 1.5]$ regardless the starting point;

3) the limit cycle equilibrium behaviors can be achieved while the decay is less than -1.5 regardless the starting point.

3.3 Number of the Concepts Influencing the Dynamic Behaviors of KMs

When concepts of KMs are fixed, KMs' dynamic behaviors also change with the adding or deleting of the relations/weights. On the other hand, deleting relations/weights may influence the number of KMs' concepts. Therefore, we only discuss how the ratios of deleting weights impact on the dynamic behaviors of the constructed KMs. The results of deleting weights from 10% to 90% are shown in Fig.5, which indicates the following conclusions that can be used to guide the experts to control or modify the dynamic behaviors of the constructed KMs:

1) starting with the fixed-points equilibrium behaviors, the limit-cycle may raise while the ratios of deleting weights are less than 40%;

2) starting with limit-cycle equilibrium behaviors, fixed-points may increase with the increase of the deleting weights' ratios;

3) starting with chaotic/oscillatory behaviors, fixed-points may increase with the increase of the deleting weights' ratios; the chaotic/oscillatory decrease with the increase of the deleting weights' ratios and the limit-cycle may arise while the ratios of the deleting weights are less than 40%.

Table 1. Terror Events Reasoning Results by Fuzzy Cognitive Map

	C_1	C_2	C_3	C_4	C_5	C_6
1	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
2	0.2913	0.2913	0.1489	0.0599	0.5154	0.4462
3	0.5343	0.3034	0.0897	−0.0745	0.9595	0.9470
4	0.4569	0.0228	−0.1113	−0.6037	0.9973	0.9974
5	−0.1907	−0.7904	−0.7190	−0.7875	0.9904	0.9964
6	−0.9772	−0.9784	−0.8278	−0.9629	−0.2065	0.8368
7	−0.9899	−0.9908	−0.8946	−0.9984	−0.9804	−0.9973
8	−0.9924	−0.9932	−0.9047	−0.3960	−1.0000	−0.9998
9	−0.9927	−0.9604	−0.5328	−0.4280	−1.0000	−0.9999
10	−0.9672	−0.8939	−0.5662	0.1002	−1.0000	−0.9999
11	−0.9669	−0.6038	0.1491	0.0503	−1.0000	−0.9998
12	−0.3303	0.2906	0.0753	0.8087	−0.9999	−0.9998
13	0.4236	0.8683	0.8376	0.7669	−0.9451	−0.9925
...						

4 Simulation and Application

4.1 Terror Events Reasoning by Fuzzy Cognitive Map

FCM1 shown in Fig.1 is constructed by an expert for describing terror events. The reasoning results of FCM1 are shown in Fig.6 and Table 1 respectively. Fig.7 is the zoom of Fig.6's anterior iterative steps. (Please refer to the inside back cover for Figs.6 and 7.) The reasoning results show that the terror events exhibit the limit-cycle equilibrium behaviors ($T \approx 20$). If the casualty and the explosion occurrence degree are very little, we know that the terror events occurrence degree is also very little. The reasoning results of the 3rd iteration (shown in Table 1) show that if the government executes very friendly foreign policy (0.9973) and very strong powers of strike (0.9974), then the occurrence degree of terror events becomes 0.4569, and the casualty and explosion occurrence degree become 0.0228 and −0.1113 respectively. Therefore, the reasoning results of the 3rd iteration are not reasonable. The 11th iteration reasoning results shows that if the government does not strike the terrorists (−0.9998), only executes wild foreign policy (−1.0000), then the terror events would hardly occur (−0.9669). This situation is also impossible to occur in real world.

4.2 Terror Event Reasoning by Knowledge Map

4.2.1 Reasoning Process

Knowledge map of KM1 is shown in Fig.2. The relation type matrix and the adjacency matrix are $\boldsymbol{R}1$ and $\boldsymbol{E}1$. The reasoning results of KM1 are shown in Fig.8 and Table 2 respectively (Please refer to the inside back cover for Fig.8). Table 3 is the step1's reasoning process of Table 2. The reasoning results of KM1 show that if the government executes very friendly foreign policy (0.9260) and very strong power of strike (0.9253), then the terror events occurrence degree is −0.0165, and the casualty and the explosion occurrence degree are −0.0058 and −0.0056 respectively. Reviewing all of the results of KM1 we can observe the tendency that with the strengthening of the friendly foreign policy and the power of strike, the terror events' occurrence degree becomes lower and lower. Compared with FCM1, the reasoning results of KM1 are more reasonable than FCM1 in emulating terror events.

Table 2. Terror Events Reasoning Results by KM1

	C_1	C_2	C_3	C_4	C_5	C_6
0	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
1	0.1000	0.1101	0.1051	0.1020	0.1000	0.1000
2	0.2050	0.0601	0.0549	0.0475	0.3154	0.2731
3	0.1868	0.0318	0.0279	0.0079	0.5357	0.5507
4	0.1371	0.0157	0.0130	−0.0183	0.7099	0.7301
5	0.0895	0.0064	0.0048	−0.0346	0.8153	0.8321
6	0.0528	0.0011	0.0002	−0.0444	0.8736	0.8859
7	0.0271	−0.0019	−0.0024	−0.0500	0.9040	0.9125
8	0.0102	−0.0037	−0.0039	−0.0531	0.9189	0.9245
9	−0.0006	−0.0047	−0.0047	−0.0547	0.9255	0.9290
10	−0.0073	−0.0052	−0.0051	−0.0554	0.9280	0.9299
11	−0.0114	−0.0055	−0.0053	−0.0558	0.9284	0.9293
12	−0.0138	−0.0057	−0.0055	−0.0559	0.9280	0.9282
...						

Table 3. First Step Reasoning Process in Table 2

	C_1	C_2	C_3	C_4	C_5	C_6
1	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
$2(P)$	0.1000	0.1050	0.1000	0.1050	0.1000	0.1000
$3(1S)$	0.1000	0.1050	0.1000	0.1020	0.1000	0.1000
$4(2S, 1D)$	0.1000	0.1101	0.1051	0.1020	0.1000	0.1000

4.2.2 Repeated Experiments on KM1

We randomly set the concepts' initial state values to view how to influence the dynamic behaviors of KM1. 50,000 times experiments have been repeated on KM1. Fig.9 shows the results of 1,000 times experiments. Fig.10 is the relations among concepts, concepts' state values and reasoning times of KM1. Fig.11 and Fig.12 are relations among repeated experiments, state values and reasoning times of the concept1 and concept6 respectively. (Please refer to the inside back cover for Figs.9–12.)

Figs.9–12 show that the random initial state values only impact on the anterior reasoning clearly, but they are little on the final results of KM1. These figures reflect that the network of KM1 can control the dynamic behaviors of KM1 and all the relations of KM1 interact in the causal relation space. Finally, they can achieve fixed-points equilibrium behaviors. These figures also show that: despite the terrorism states in real world, the terror events would have little probability to occur if we execute a friendly foreign policy and a strong power strike on terrorism.

4.2.3 Stability of KM1

The results of 1,000 times repeated stability experiments (with different initial state values of concepts) on the reasoning process of KM1 are shown in Fig.13. This figure indicates that the disorders of all the concept's outputs of KM1 are controlled by itself and the final steady

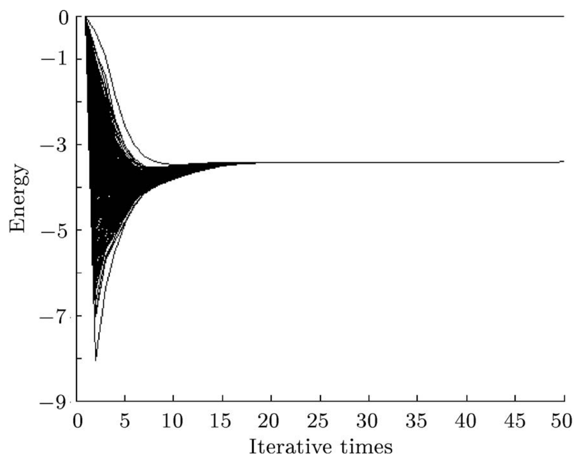

Fig.13. 1,000 times repeated stability experiments on the reasoning process of KM1.

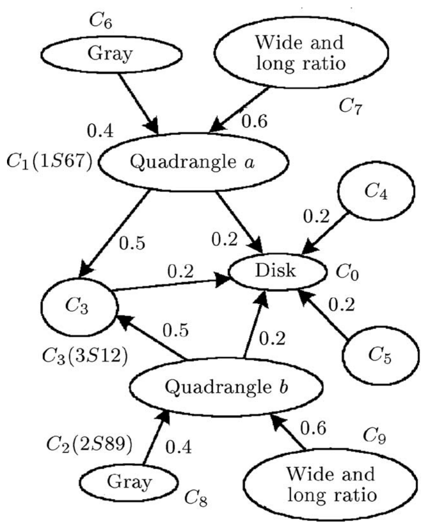

Fig.14. KM of the disk recognition (denoted as KM2). (C_3 represents the contain degree between quadrangle a and b; C_4 represents the parallel degree between horizontal lines; C_5 represents the parallel degree between vertical lines).

(a) (b) (c) (d) (e)

Fig.15. Results of the disk recognition by using knowledge map. (a) Original image. (b) Results of image after edge extraction. (c) Image of long lines. (d) Basic shapes of the disk. (e) Recognition results of the disk by KM2.

states of KM1 can be achieved. But in some KMs, different initial state values of concepts can achieve different states, depending on the structure of KMs.

4.3 Application on Disk Recognition

The knowledge of disk recognition is represented by the KM shown in Fig.14 (denoted as KM2), which is difficult to be represented by FCM.

The relation and the adjacency matrix of KM2 correspond to $\boldsymbol{R}2$ and $\boldsymbol{E}2$ respectively. After the reasoning of KM2, the reasoning results are shown in Fig.15 and the value of C_0 is 0.78. From Fig.15 we know that KM2 can recognize the disk correctly.

$$\boldsymbol{R}2 = \begin{vmatrix} N & N & N & N & N & N & N & N & N & N \\ R_{sc} & N & N & 3S & N & N & N & N & N & N \\ R_{sc} & N & N & 3S & N & N & N & N & N & N \\ R_{sc} & N & N & N & N & N & N & N & N & N \\ R_{sc} & N & N & N & N & N & N & N & N & N \\ R_{sc} & N & N & N & N & N & N & N & N & N \\ N & 1S & N & N & N & N & N & N & N & N \\ N & 1S & N & N & N & N & N & N & N & N \\ N & N & 2S & N & N & N & N & N & N & N \\ N & N & 2S & N & N & N & N & N & N & N \end{vmatrix}$$

$$\boldsymbol{E}2 = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}.$$

5 Analyses

The KMs have the following capabilities over FCMs in emulating real world.

1) KMs consider not only simple cause-effect relations, but also sequential, time-delay, and conditional probabilistic causal relations at the same time when emulating real world.

2) The effect concept's state value of KM at time $(t+1)$ depends on not only causal concepts' state values at time t but also its own actual state value. FCMs only consider the actual causal relations; they do not include the actual state of the effect concept. So, the reasoning values of FCMs often change sharply. This does not accord with real world. According to Rule 5, the value of the next effect concept of KM includes not only the actual state values of the causal concepts but also its own actual value.

3) If a concept does not have any causal concept, it can maintain its own original value in the reasoning process of KMs. In FCMs, if a concept does not have any causal concept, the next reasoning value of the concept would be zero according to the reasoning rules of FCMs. In the reasoning process of KMs, concepts can keep their original values in the reasoning process of KMs according to Rule 3, so KMs can avoid this type of distortion in the reasoning process.

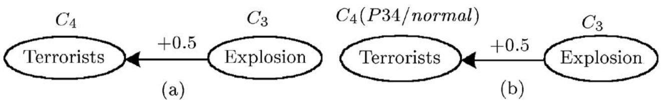

Fig.16. Different representation of relations between terrorists and explosion. (a) Relation between terrorists and explosion represented by FCM. (b) Relation between terrorists and explosion represented by KM.

4) Before the reasoning of the simple cause-effect relations, time-delay causal relations and sequential relations, conditional probabilistic causal relations can be reasoned. In FCMs, if one concept's occurrence increases/decreases another concept's existent degree, then the increased/decreased value would be used in the reasoning process. For example, we know that there may be terrorists in a town (assuming that its existent degree is 0.6), if one day we hear from news that the town was attacked by explosion (shown in Fig.16). At that moment, the terrorists' existent degree may jump to a high value. Taking Figs.16(a) and 16(b) as an example, assume that $V_{C_4}(0) = V_{C_3}(0) = 0.6$. FCM reasoning out: $V_{C_4}(1) = f(w_{ij} \times V_{C_3}(0)) = \tanh(0.5 \times 0.6) = 0.2913$; KM reasoning out: $V_{C_4}(1) = V_{C_4}(0) + u(w_{34} \times V_{C_3}(0)) = 0.8913$. Common sense tells us: when an explosion occurs, the terrorists' existent degree may increase. The reasoning results of FCM are contrary. Therefore, the reasoning of KM is more reasonable than FCM.

6 Conclusions

Fuzzy cognitive maps only describe simple cause-effect relations of causal knowledge. The knowledge map model can describe not only simple cause-effect relations, but also the time-delay causal relations, conditional probabilistic causal relations, and sequential relations. The mathematical model and dynamic behaviors of KMs are discussed. The experiments on the dynamic behaviors of KMs show that under certain conditions, their behaviors can be translated between fixed-points, limit-cycle or chaos/oscillation. And those results can guide experts to control/modify the dynamic behaviors of the constructed KMs that do not behave in accordance with their expectation. The results of the terror event simulations and disk recognition indicate that knowledge maps can emulate real world more natural and powerful than FCMs. But the experiments method is qualitative and the model still needs further verification in practical applications. Ongoing works are to find new method to measure the dynamic behaviors of KMs and use knowledge maps to realize knowledge sharing, capturing and understanding in real Knowledge Grid environments[17].

References

[1] Stylios C D, Groumpos P P. Fuzzy cognitive maps: A soft computing technique for intelligent control. In *Proc. IEEE Int. Symp. Intelligent Control,* Patras, 2000, pp.97–102.

[2] Miao Y, Liu Z Q. On causal inference in fuzzy cognitive maps. *IEEE Trans. Fuzzy System,* 2000, 8(1): 107–119.

[3] Miao Y, Liu Z Q *et al.* Dynamic cognitive network. *IEEE Trans. Fuzzy System,* 2001, 9(5): 760–770.

[4] Hagiwara M. Extended fuzzy cognitive maps. In *Proc. IEEE Int. Conf. Fuzzy System,* San Diego, 1992, pp.795–801.

[5] Kosko B. Fuzzy Engineering. Prentice Hall, 1997.

[6] Chaib-draa B. Causal maps: Theory, implementation and practical applications in multi-agent environments. *IEEE Trans. Knowledge and Data Engineering,* 2002, 14(6): 1–17.

[7] Pelaez C E, Bowles J B. Applying fuzzy cognitive maps knowledge representation to failure modes effects analysis. In *Proc. Annual Reliability and Maintainability Symposium,* 1995, pp,450–456.

[8] Liu Z Q, Satur R. Contextual fuzzy cognitive maps for decision support in geographic information systems. *IEEE Trans. Fuzzy Systems,* 1999, 7(10): 495–502.

[9] Satur R, Liu Z Q. A contextual fuzzy cognitive maps framework for geographic information systems. *IEEE Trans. Fuzzy Systems,* 1999, 7(10): 481–494.

[10] Noha J B, Lee K C. A case-based reasoning approach to cognitive maps-driven tacit knowledge management. *Expert Systems with Applications,* 2000, 19: 249–259.

[11] Groumpos P P, Stylios C D. Modeling supervisory control systems using fuzzy cognitive maps. *Chaos Solitons and Fractals,* 2000, 11: 329–336.

[12] Stylios C D, Groumpos P P. The challenge of modeling supervisory systems using fuzzy cognitive maps. *Journal of Intelligent Manufacturing,* 1999, 9: 339–345.

[13] Dickerson J A, Kosko B. Virtual worlds as fuzzy dynamical systems. *Spring,* 1994, 3(2): 173–189.

[14] Leea K C, Lee S A. Cognitive maps simulation approach to adjusting the design factors of the electronic commerce web sites. *Expert Systems with Applications,* 2003, 24: 1–11.

[15] Zhuge H, Luo X F. Knowledge map model. *The Third Int. Conf. Grid and Cooperative Computing (GCC2004), Lecture Notes in Computer Science 3251/2004,* pp.381–388.

[16] Obata T, Hagiwara M. Neural cognitive maps. http://citeseer.ist.psu.edu/.

[17] Zhuge H. China's E-Science Knowledge Grid Environment. *IEEE Intelligent Systems,* 2004, 19(1): 13–17.

Hai Zhuge is a professor and director of the Key Lab of Intelligent Information Processing at ICT, CAS, and the founder of the China Knowledge Grid Research Group (http://kg.ict.ac.cn). Currently, he is the chief scientist of the China National Semantic Grid Project. He was keynote speaker at several International Conferences. He was chair of the 2nd International Workshop on Knowledge Grid and Grid Intelligence and program co-chair of the 4th International Conference on Grid and Cooperative Computing (GCC2005). He organized several special issues on Knowledge Grid and Semantic Grid for international journals. He is serving as area editor of the *Journal of Systems and Software* and *Journal of Computer Science and Technology*, associate editor of *Future Generation Computer Systems*, and editorial member of the *Information and Management* and the *Electronic Commerce Research and Applications.* His current research interest is model and theory on the future interconnection environment and applications in China. His monograph "*The Knowledge Grid*" is the first book in the area. He is the author of over ninety papers appeared mainly in leading international conferences and journals. He is listed in the Top 15 software engineering and system scholars in an assessment report published in Journal of Systems and Software in 2005. He is a senior member of IEEE and a member of ACM.

Xiang-Feng Luo's research fields include knowledge capturing, semantic grid and knowledge grid, artificial intelligence and pattern recognition. Currently he is in charge of a research project supported by the National Natural Science Foundation of China; a research project supported by Innovation Fund of Chinese Academy of Science, and participated several research projects supported by national and provincial science funds.

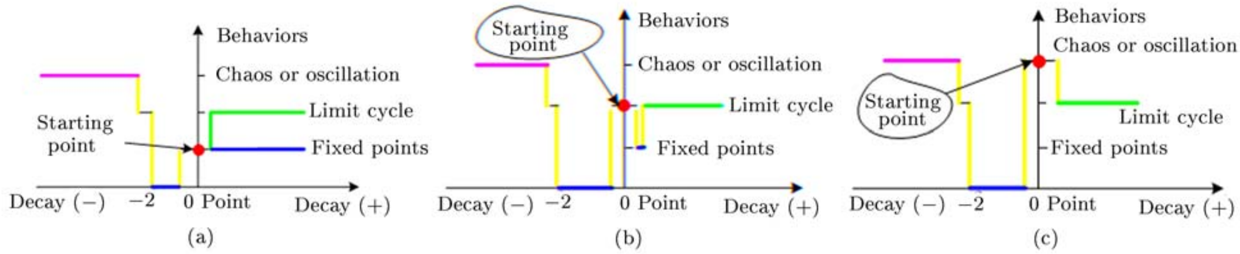

Fig.3. Dynamic behaviors of KMs while weights change with (1) (the red circle stands for the starting point, green lines stand for limit cycle behavior, blue lines stand for fixed point behavior, pink lines stand for chaos/oscillation behavior and yellow lines stand for the transition of state. The meanings of the color lines of the following figures have the same meaning as in this figure). (a) Dynamic behaviors of KMs starting with the fixed-points equilibrium behaviors. (b) Dynamic behaviors of KMs starting with limit-cycle equilibrium behaviors. (c) Dynamic behaviors of KMs starting with chaotic/oscillatory behaviors.

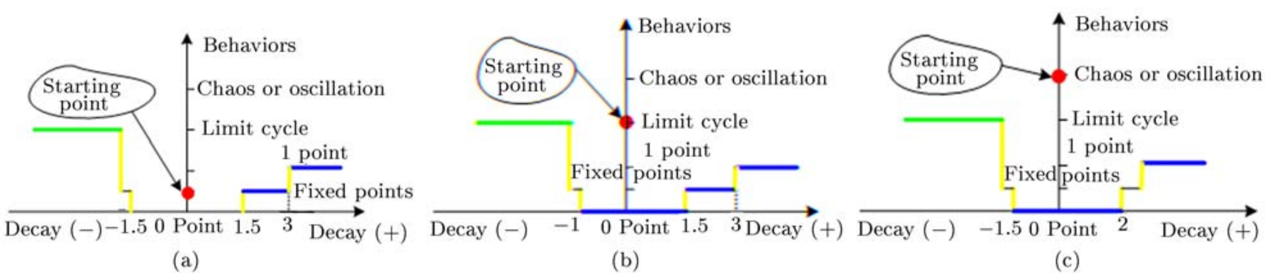

Fig.4. Dynamic behaviors of KMs while weights change with (2). (a) Dynamic behaviors of KMs starting with fixed-points equilibrium behaviors. (b) Dynamic behaviors of KMs starting with limit-cycle equilibrium behaviors. (c) Dynamic behaviors of KMs starting with chaotic/oscillatory behaviors.

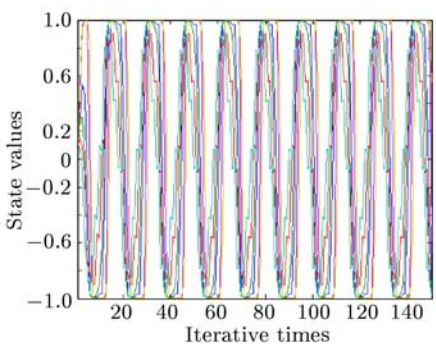

Fig.6. Terror events reasoning results by fuzzy cognitive map.

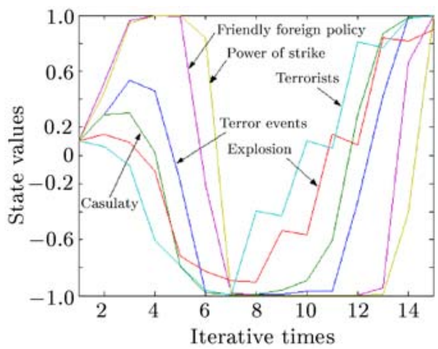

Fig.7. Zoom of Fig.6's anterior iterative times.

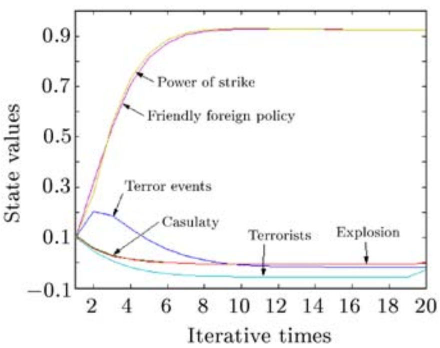

Fig. 8. Terror events reasoning results by knowledge map.

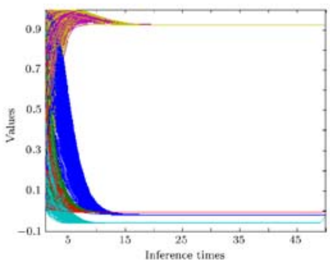

Fig.9. 1,000 times repeated experiments on KM1 using random initial state values of concepts.

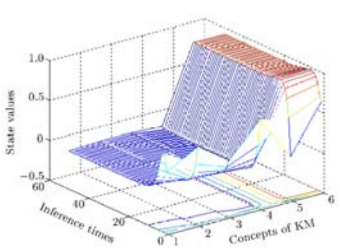

Fig.10. Relations among concepts, state values, and reasoning times of KM1.

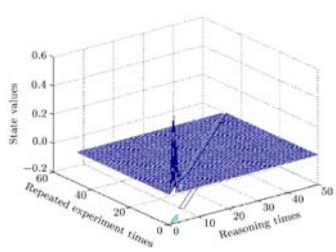

Fig.11. Relations among repeated experiment times, state values, and reasoning times on concept1.

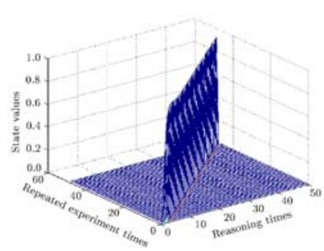

Fig.12. Relations among repeated reasoning times, state values, and experiment times on concept 6.